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Binary spreading process with parity conservation

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Recently there has been a debate concerning the universal properties of the phase transition in the pair contact process with diffusion (PCPD) $2A \to 3A$, $2A \to 0$. Although some of the critical exponents seem to coincide with those of the so-called parity-conserving universality class, it was suggested that the PCPD might represent an independent class of phase transitions. This point of view is motivated by the argument that the PCPD does not conserve parity of the particle number. In the present work we question what happens if the parity conservation law is restored. To this end, we consider the reaction-diffusion process $2A \to 4A$, $2A \to 0$. Surprisingly, this process displays the same type of critical behavior, leading to the conclusion that the most important characteristics of the PCPD is the use of binary reactions for spreading, regardless of whether parity is conserved or not.

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In the field of nonequilibrium critical phenomena, the study of phase transitions from fluctuating into absorbing states continues to attract considerable attention [1]. It is believed that phase transitions into absorbing states can be categorized into a finite number of universality classes characterizing the long-range properties at the critical point.

So far two universality classes are firmly established. The first and most prominent one is the universality class of directed percolation (DP) [2,3], which describes the spreading of particles according to the reaction diffusion scheme

$$A \xrightarrow{\lambda} 2A, \quad A \xrightarrow{\mu} 0, \tag{1}$$

where λ and μ are the rates for offspring production and particle decay, respectively. In addition, particles are allowed to diffuse and the maximal density of particles is limited. Therefore, if λ is sufficiently high, the system is in a fluctuating (active) high-density phase, while for low values of λ it reaches the (inactive) vacuum state within exponentially short time.

The second established universality class is the so-called parity-conserving (PC) class of phase transitions [4–6], which appear in spreading processes with parity-conserving dynamics such as

$$A \to 3A, \quad 2A \to 0. \tag{2}$$

In this type of spreading process, particles can only annihilate in pairs so that the absorbing phase is characterized by an *algebraic* decay of the particle density with time. In one spatial dimension, parity conservation allows the particles to be considered as kinks between oppositely oriented domains [7,8]. Using this interpretation, the process can be regarded as a directed percolation process with two Z_2-symmetric absorbing states (DP2) [9]. To some extent, the situation is similar to the one in the kinetic Ising model, although in the present case the transition is generated by *interfacial noise*, instead of bulk noise [10].

Apart from these two established universality classes, there are only few other possible candidates, the most mysterious being the pair contact process with diffusion (PCPD), sometimes also called annihilation-fission process. This process was originally introduced by Howard and Täuber as a model interpolating between ''real'' and ''imaginary'' noise [11] and corresponds to the reaction-diffusion scheme

$$2A \to 3A, \quad 2A \to 0. \tag{3}$$

Interestingly, this model exhibits a nontrivial phase transition even in one spatial dimension. As in the PC class, particles can only annihilate in pairs, so that the particle density in the inactive phase decays algebraically. Moreover, the model has two absorbing states, namely, the empty lattice and the state with a single diffusing particle. Because of these similarities and an apparent numerical coincidence of certain critical exponents, Carlon *et al.* raised the possibility that the transition in the PCPD might belong to the PC universality class [12]. A different point of view was presented in Ref. [13], suggesting that the broken parity conservation law in the reaction diffusion scheme (3) should drive the system away from the PC class, leading to the conjecture that the transition in the PCPD might belong to a novel, yet unexplored universality class.

Subsequent high-precision simulations [14] confirmed that some of the critical exponents, especially the order parameter exponent β, seem to be incompatible with the PC hypothesis, supporting the viewpoint of Ref. [13]. On the other hand, the simulations revealed unexpected difficulties, in particular unusually strong corrections to scaling. It turned out that even after 10^7 time steps it is not yet clear whether the ''true'' scaling regime has already been reached. Therefore, the critical exponents in one spatial dimension could only be determined with considerable uncertainty (see Table I).

Concerning the PCPD transition, there are many open questions: Do the critical properties depend on the details of the dynamics or do they indeed represent an independent

TABLE I. Estimates of the critical exponents for directed percolation, the parity-conserving class, and various binary spreading processes.

Class	β	$\nu_\perp$	$\nu_\parallel$
DP	0.2765	1.0969	1.734
PC	0.92(2)	1.83(3)	3.22(6)
PCPD	< 0.6	1.0 . . . 1.2	1.8 . . . 2.1
Cyclic model	0.38(6)	1.0(1)	1.8(1)
Present work	0.50(5)	1.17(7)	2.1(1)

universality class that has not been investigated before? Does the simple scaling picture, which involves only a single length scale, still apply or is it necessary to consider the possibility of multiscaling? What is the origin of the scaling corrections and what are the precise values of the critical exponents?

A possible phenomenological explanation of the transition in the PCPD was proposed in Ref. [15]. This explanation is based on the assumption that the most salient features of the process are the interplay of (a) diffusing solitary particles, and (b) spreading when at least two particles meet at neighboring sites. It was conjectured that a cyclically coupled model with two particle species consisting of a DP process and an annihilation process should display the same critical behavior as the PCPD. In fact, numerical estimates of the critical exponents seem to be compatible with the PCPD results. However, the mere numerical coincidence within rather large error bars cannot yet be regarded as a proof. Therefore, it would be interesting to investigate the problem by alternative methods such as real-space renormalization [16].

Since in Ref. [13] the main argument against the PC hypothesis has been the broken parity conservation law in the PCPD, it would be interesting to find out what happens if the conservation law is restored. This can be done by modifying the particle creation process in the reaction-diffusion scheme (3), e.g., by considering the process

$$2A \to 4A, \quad 2A \to 0. \tag{4}$$

In this process, the number of particles is conserved modulo 2. As a surprising result, which will be presented below, we find that this modification does not change the type of critical behavior at the transition, i.e., the process still behaves in the same way as the PCPD, as already observed in the corresponding bosonic field theory [11]. Thus, in the attempt to understand the physics of the PCPD, it would be misleading to focus exclusively on the parity conservation law, rather it is more important whether we are dealing with a *unary* or a *binary* spreading process. In a unary spreading process (e.g., in DP and PC models), a *single* particle is able to produce one or several offspring. On the contrary, in a binary spreading process such as the PCPD, two particles are required to meet at the same or neighboring sites in order to generate offspring.

Definition of the model. The process defined in Eq. (4), which will be studied in the present work, is a binary spreading process with parity-conserving dynamics. It is defined on a one-dimensional lattice with L sites and periodic boundary conditions, where local variables $s_i = 0,1$ indicate whether the site is empty or occupied by a particle. The model is controlled by a single parameter p and evolves by random-sequential updates according to the following dynamic rules. For each update a site i is randomly selected and a random number $z \in (0,1)$ is drawn from a flat distribution. Then the following moves are carried out:

(i) If $p < z$ and site i is occupied by a particle, it hops randomly to the left or to the right. If the selected target site is already occupied, both particles annihilate instantaneously.

(ii) If $p > z$ and the two sites i and $i+1$ are occupied, this pair of particles generates two offspring to the left (sites $i-2, i-1$) or to the right (sites $i+2, i+3$) with equal probability. If the generated particles land on an already occupied site, they annihilate instantaneously.

As usual, L update attempts correspond to a time increment of 1. The dynamic rules given above can also be defined in terms of a reaction-diffusion scheme

$$\begin{array}{lll}
0A \leftrightarrow A0 & \text{at rate} & p/2 \\
AA \to 00 & & p \\
AA00 \leftrightarrow AAAA & & q/2 \\
00AA \leftrightarrow AAAA & & q/2 \\
AAA0 \leftrightarrow AA0A & & q/2 \\
0AAA \leftrightarrow A0AA & & q/2,
\end{array}$$

where $q = 1 - p$.

Numerical analysis. In order to estimate the critical exponents characterizing the transition between the active and the absorbing phase, we performed standard Monte Carlo simulations. To this end we measured the density of particles, $\rho(t) = 1/L \sum_i s_i(t)$, starting with a fully occupied lattice as initial condition. At the critical point, this quantity should decay algebraically as $\rho(t) \sim t^{-\delta}$. Using this criterion (see Fig. 1) we estimated the critical point by $p_c = 0.0895(2)$. For the decay exponent we obtain the estimate

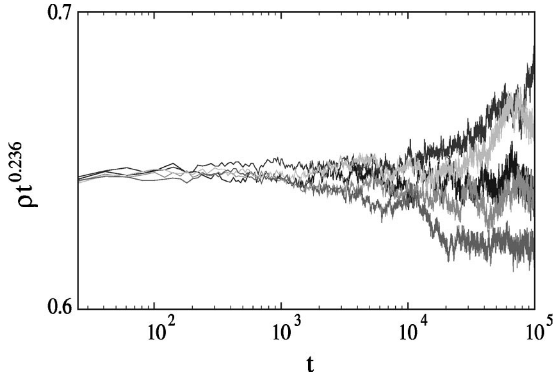

FIG. 1. The density of particles, $\rho(t)$, times $t^{0.236}$ as a function of time for $p = 0.0893$, 0.0894, 0.0895, 0.0896, and 0.0897 from top to bottom, averaged over 2000 runs on a system with 2048 sizes.

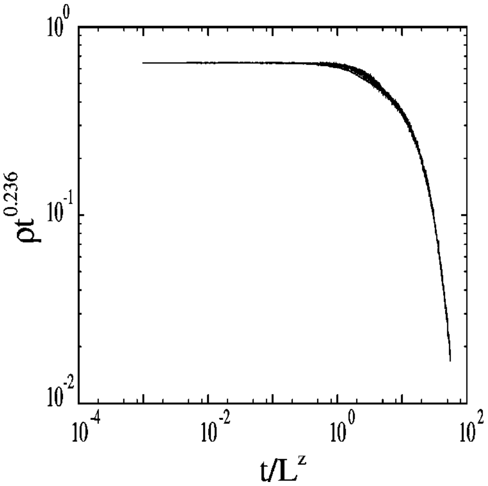

FIG. 2. Finite-size data collapse according to Eq. (6) for system sizes $L=64$, 90, 128, 180, and 256 averaged over 50 000 runs.

$$\delta=\beta/\nu_{\parallel}=0.236(10). \tag{5}$$

Next, in order to determine the dynamic exponent $z=\nu_{\parallel}/\nu_{\perp}$, we performed finite-size simulations at the critical point. Here the density of particles should obey the following finite-size scaling form:

$$\rho(t,L)\sim t^{-\delta}f(t/L^z), \tag{6}$$

where f is a universal scaling function. Using the previous estimate $\delta=0.236$, the best collapse is obtained for $z=1.80(5)$ (see Fig. 2). Similarly, the third independent exponent $\nu_{\parallel}$ can be determined by studying the behavior of the density of particles below and above criticality. Here we expect the scaling form

$$\rho(t,\epsilon)\sim t^{-\delta}g(t\epsilon^{\nu_{\parallel}}), \tag{7}$$

where $\epsilon=|p-p_c|$ denotes the distance from the critical point. Using the estimate $\delta=0.236$, the best collapse is obtained for $\nu_{\parallel}=2.1(1)$ (see Fig. 3). Combining these estimates we arrive at the result

$$\beta=0.50(5), \quad \nu_{\perp}=1.17(7), \quad \nu_{\parallel}=2.1(1). \tag{8}$$

As an additional test, we performed dynamic simulations starting with a seed of a single pair of particles located in the center, measuring the survival probability $P(t)$ that the system has not yet reached an absorbing states, the average number of particles $N(t)$, and the mean square spreading from the origin $R^2(t)$ averaged over the survival runs. At criticality these quantities should obey the power laws $P(t)\sim t^{-\delta'}$, $N(t)\sim t^{\eta}$, and $R^2(t)\sim t^{2/z}$, where δ' and η are dynamical exponents. Measuring these quantities we observe strong corrections to scaling, which make it impossible to obtain reliable estimates for the critical exponents. Fitting straight lines over the last decade we find the values (see Fig. 4)

$$\delta'\approx 0.1, \quad \eta\approx 0.2, \quad 2/z\approx 1.15, \tag{9}$$

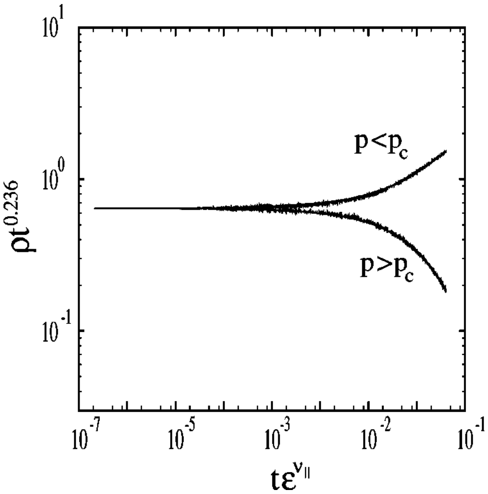

FIG. 3. Data collapse for off-critical simulations according to the scaling form (7) for $\epsilon=0.0001$, 0.0002, . . . , 0.0064 averaged over 2000 runs.

without being able to estimate the errors. Nevertheless the estimate for $2/z$ is in rough agreement with the previous estimate $z=1.80(2)$.

We also measured the density of *pairs* of particles, which can be used as an alternative order parameter in the present model. Here we find the same type of critical behavior, although with slightly different estimates for the critical exponents.

Discussion. As shown in Table I, our estimates for the critical exponents are in fair agreement with those of the PCPD. In particular, we can rule out the possibility of a DP or a PC transition. This result is surprising, since it suggests that we can introduce an additional symmetry without changing the critical behavior of the transition. This means that parity conservation is *irrelevant* for the long-range properties at the transition.

To understand this observation, we note that there is another well-known example where parity conservation is irrelevant, namely, the annihilation process $2A\rightarrow 0$ in comparison to the coagulation process $2A\rightarrow A$. Both processes are known to belong to the same universality class and can even be related by an exact similarity transformation [17]. This is due to the fact that the even and the odd sector in the parity-

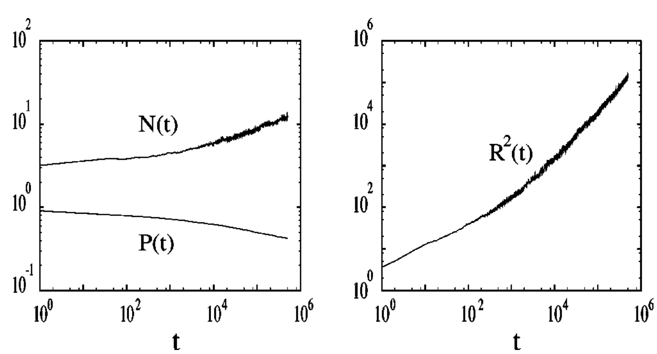

FIG. 4. The survival probability $P(t)$, the average number of particles $N(t)$, and the mean square spreading $R^2(t)$ starting with a single pair of particles.

conserving process $2A \rightarrow 0$ are essentially equivalent, since in both cases the particle density decays algebraically until the system is trapped in an absorbing state (namely, the empty lattice or a state with a single diffusing particle). Breaking the parity conservation law by a weak perturbation, the system begins to switch between the even and the odd sector. However, this ongoing switching process does not change the universal behavior since, from a macroscopic point of view, the physical properties of both sectors cannot be distinguished. In the present model the situation is quite similar. In both sectors we have a transition from an active phase into an absorbing state. Therefore, the physical properties of both sectors are essentially the same so that the breakdown of parity conservation does not change the critical behavior.

In the PC class, however, the situation is completely different. In this case parity conservation is indeed relevant. For example, in the branching-annihilating random walk with even number of offspring $A \rightarrow 3A$, $2A \rightarrow 0$ the two sectors are not equivalent because only one of them has an absorbing state. Therefore, even a tiny violation of the conservation law drives the transition away from the PC class.

How can we verify whether parity conservation in a given system is relevant or not? One way would be to investigate how the critical behavior changes if the symmetry is broken. Another much more elegant method would be to compare seed simulations in the even and the odd sector (i.e., starting with two or three particles). Here the survival probability $P(t)$ has to be defined as the probability that there are at least two particles left. If the survival exponent δ' and the critical initial slip exponent η are different in both sectors (as they are in the case of the PC class), parity conservation is relevant. However, if the exponents do not depend on the sector (as in the present model), we expect the parity symmetry to be irrelevant.

In the light of these results, the conjecture of Ref. [13] has to be refined. It is true that we cannot have PC critical behavior in systems without parity conservation or an equivalent Z_2 symmetry. On the other hand, the broken parity conservation law is not the main characteristic of the transition in the PCPD; rather, it is possible to restore this symmetry without changing the critical behavior. Therefore, a necessary condition for existence of this class, for which our understanding is still incomplete, seems to be the *binary* nature of the process for offspring production, i.e., two particles have to meet at the same place in order to create new particles.

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